1.3 Use Midpoint and Distance Formulas

Before You found lengths of segments.

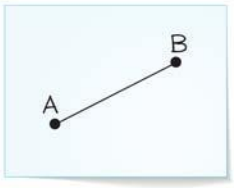
Now You will find lengths of segments in the coordinate plane.

Why? So you can find an unknown length, as in Example 1.

Key Vocabulary
- **midpoint**
- **segment bisector**

ACTIVITY FOLD A SEGMENT BISECTOR

STEP 1

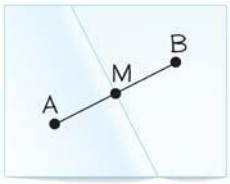


Draw $\overline{AB}$ on a piece of paper.

STEP 2

Fold the paper so that *B* is on top of *A*.

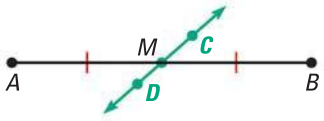
STEP 3



Label point *M*. Compare *AM*, *MB*, and *AB*.

MIDPOINTS AND BISECTORS The **midpoint** of a segment is the point that divides the segment into two congruent segments. A **segment bisector** is a point, ray, line, line segment, or plane that intersects the segment at its midpoint. A midpoint or a segment bisector *bisects* a segment.

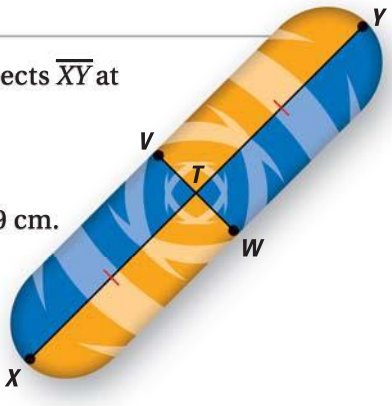
***M* is the midpoint of $\overline{AB}$.**
So, $\overline{AM} \cong \overline{MB}$ and *AM* = *MB*.



$\overleftrightarrow{CD}$ is a segment bisector of $\overline{AB}$.
So, $\overline{AM} \cong \overline{MB}$ and *AM* = *MB*.

EXAMPLE 1 Find segment lengths

SKATEBOARD In the skateboard design, $\overline{VW}$ bisects $\overline{XY}$ at point *T*, and $XT = 39.9$ cm. Find XY.



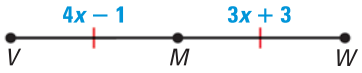
Solution

Point *T* is the midpoint of $\overline{XY}$. So, $XT = TY = 39.9$ cm.

$XY = XT + TY$	**Segment Addition Postulate**
$= 39.9 + 39.9$	**Substitute.**
$= 79.8$ cm	**Add.**

EXAMPLE 2 Use algebra with segment lengths

ALGEBRA Point M is the midpoint of $\overline{VW}$. Find the length of $\overline{VM}$.



Solution

REVIEW ALGEBRA
For help with solving equations, see p. 875.

STEP 1 **Write** and solve an equation. Use the fact that that $VM = MW$.

$VM = MW$	Write equation.
$4x - 1 = 3x + 3$	Substitute.
$x - 1 = 3$	Subtract $3x$ from each side.
$x = 4$	Add 1 to each side.

STEP 2 **Evaluate** the expression for VM when $x = 4$.

$VM = 4x - 1 = 4(4) - 1 = 15$

▶ So, the length of $\overline{VM}$ is 15.

CHECK Because $VM = MW$, the length of $\overline{MW}$ should be 15. If you evaluate the expression for MW, you should find that $MW = 15$.

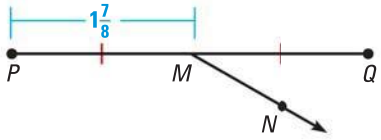
$MW = 3x + 3 = 3(4) + 3 = 15$ ✓

GUIDED PRACTICE for Examples 1 and 2

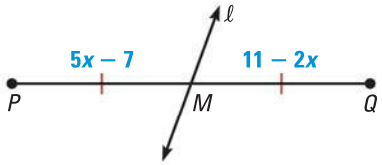
READ DIRECTIONS
Always read direction lines carefully. Notice that this direction line has two parts.

In Exercises 1 and 2, identify the segment bisector of $\overline{PQ}$. Then find PQ.

1.



2.



COORDINATE PLANE You can use the coordinates of the endpoints of a segment to find the coordinates of the midpoint.

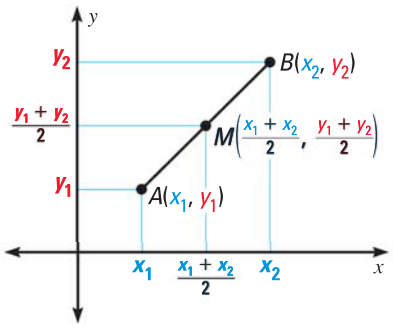
KEY CONCEPT For Your Notebook

The Midpoint Formula

The coordinates of the midpoint of a segment are the averages of the x-coordinates and of the y-coordinates of the endpoints.

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the midpoint M of $\overline{AB}$ has coordinates

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$



EXAMPLE 3 Use the Midpoint Formula

a. FIND MIDPOINT The endpoints of $\overline{RS}$ are $R(1, -3)$ and $S(4, 2)$. Find the coordinates of the midpoint M.

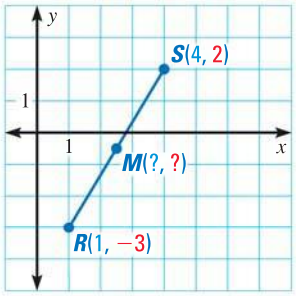
b. FIND ENDPOINT The midpoint of $\overline{JK}$ is $M(2, 1)$. One endpoint is $J(1, 4)$. Find the coordinates of endpoint K.

Solution

a. FIND MIDPOINT Use the Midpoint Formula.

$$M\left(\frac{1+4}{2}, \frac{-3+2}{2}\right) = M\left(\frac{5}{2}, -\frac{1}{2}\right)$$

▶ The coordinates of the midpoint M are $\left(\frac{5}{2}, -\frac{1}{2}\right)$.



b. FIND ENDPOINT Let (x, y) be the coordinates of endpoint K. Use the Midpoint Formula.

CLEAR FRACTIONS Multiply each side of the equation by the denominator to clear the fraction.

STEP 1 Find x.

$$\frac{1+x}{2} = 2$$

$$1 + x = 4$$

$$x = 3$$

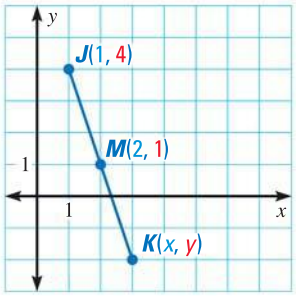
STEP 2 Find y.

$$\frac{4+y}{2} = 1$$

$$4 + y = 2$$

$$y = -2$$

▶ The coordinates of endpoint K are $(3, -2)$.



GUIDED PRACTICE for Example 3

3. The endpoints of $\overline{AB}$ are $A(1, 2)$ and $B(7, 8)$. Find the coordinates of the midpoint M.

4. The midpoint of $\overline{VW}$ is $M(-1, -2)$. One endpoint is $W(4, 4)$. Find the coordinates of endpoint V.

DISTANCE FORMULA The Distance Formula is a formula for computing the distance between two points in a coordinate plane.

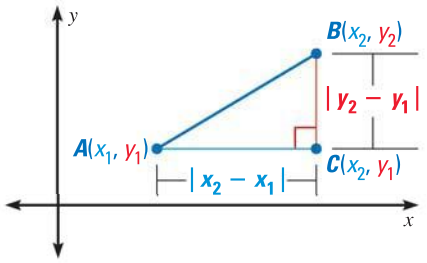
KEY CONCEPT *For Your Notebook*

The Distance Formula

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are points in a coordinate plane, then the distance between A and B is

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

READ DIAGRAMS The red mark at one corner of the triangle shown indicates a right triangle.



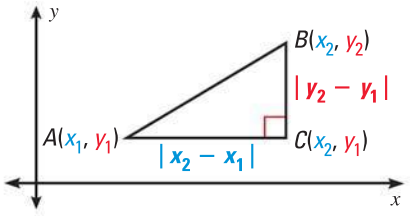
The Distance Formula is based on the *Pythagorean Theorem*, which you will see again when you work with right triangles in Chapter 7.

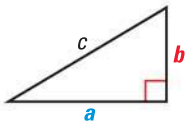
Distance Formula

$$(AB)^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

Pythagorean Theorem

$$c^2 = a^2 + b^2$$





EXAMPLE 4 Standardized Test Practice

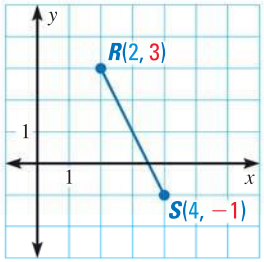
ELIMINATE CHOICES
Drawing a diagram can help you eliminate choices. You can see that choice A is not large enough to be *RS*.

What is the approximate length of $\overline{RS}$ with endpoints $R(2, 3)$ and $S(4, -1)$?

Ⓐ 1.4 units Ⓑ 4.0 units Ⓒ 4.5 units Ⓓ 6 units

Solution

Use the Distance Formula. You may find it helpful to draw a diagram.



$$RS = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ **Distance Formula**

$$= \sqrt{[(4 - 2)]^2 + [(-1) - 3]^2}$$ **Substitute.**

$$= \sqrt{(2)^2 + (-4)^2}$$ **Subtract.**

$$= \sqrt{4 + 16}$$ **Evaluate powers.**

$$= \sqrt{20}$$ **Add.**

READ SYMBOLS
The symbol ≈ means "is approximately equal to."

$$\approx 4.47$$ **Use a calculator to approximate the square root.**

▶ The correct answer is C. Ⓐ Ⓑ Ⓒ Ⓓ

GUIDED PRACTICE for Example 4

5. In Example 4, does it matter which ordered pair you choose to substitute for (x_1, y_1) and which ordered pair you choose to substitute for (x_2, y_2)? *Explain.*

6. What is the approximate length of $\overline{AB}$, with endpoints $A(-3, 2)$ and $B(1, -4)$?

Ⓐ 6.1 units Ⓑ 7.2 units Ⓒ 8.5 units Ⓓ 10.0 units

1.3 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 35, and 49

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 23, 34, 41, 42, and 53

SKILL PRACTICE

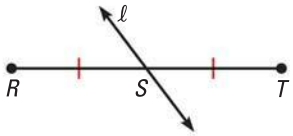
1. VOCABULARY Copy and complete: To find the length of $\overline{AB}$, with endpoints $A(-7, 5)$ and $B(4, -6)$, you can use the ___?___.

2. ★ WRITING *Explain* what it means to bisect a segment. Why is it impossible to bisect a line?

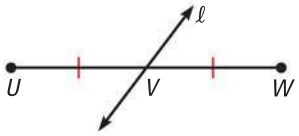
EXAMPLE 1 on p. 15 for Exs. 3–10

FINDING LENGTHS **Line ℓ bisects the segment. Find the indicated length.**

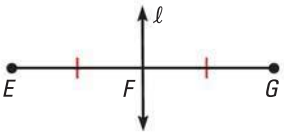
3. Find RT if $RS = 5\frac{1}{8}$ in.



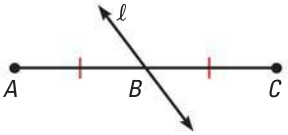
4. Find UW if $VW = \frac{5}{8}$ in.



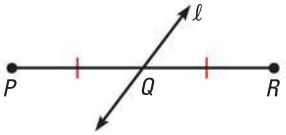
5. Find EG if $EF = 13$ cm.



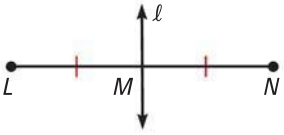
6. Find BC if $AC = 19$ cm.



7. Find QR if $PR = 9\frac{1}{2}$ in.



8. Find LM if $LN = 137$ mm.



9. SEGMENT BISECTOR Line RS bisects $\overline{PQ}$ at point R. Find RQ if $PQ = 4\frac{3}{4}$ inches.

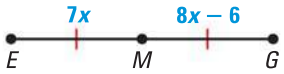
10. SEGMENT BISECTOR Point T bisects $\overline{UV}$. Find UV if $UT = 2\frac{7}{8}$ inches.

EXAMPLE 2 on p. 16 for Exs. 11–16

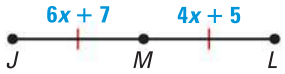
xy ALGEBRA **In each diagram, M is the midpoint of the segment. Find the indicated length.**

11. Find AM.

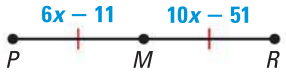
12. Find EM.



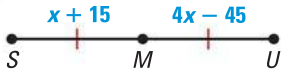
13. Find JM.



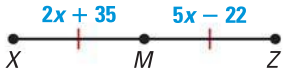
14. Find PR.



15. Find SU.



16. Find XZ.



EXAMPLE 3 on p. 17 for Exs. 17–30

FINDING MIDPOINTS **Find the coordinates of the midpoint of the segment with the given endpoints.**

17. $C(3, 5)$ and $D(7, 5)$

18. $E(0, 4)$ and $F(4, 3)$

19. $G(-4, 4)$ and $H(6, 4)$

20. $J(-7, -5)$ and $K(-3, 7)$

21. $P(-8, -7)$ and $Q(11, 5)$

22. $S(-3, 3)$ and $T(-8, 6)$

23. ★ WRITING Develop a formula for finding the midpoint of a segment with endpoints $A(0, 0)$ and $B(m, n)$. *Explain* your thinking.

24. ERROR ANALYSIS *Describe* the error made in finding the coordinates of the midpoint of a segment with endpoints $S(8, 3)$ and $T(2, -1)$.

$$\left(\frac{8-2}{2}, \frac{3-(-1)}{2}\right) = (3, 2)$$ ✗

FINDING ENDPOINTS **Use the given endpoint R and midpoint M of $\overline{RS}$ to find the coordinates of the other endpoint S.**

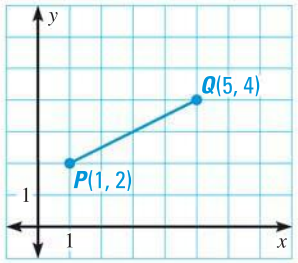
25. $R(3, 0)$, $M(0, 5)$ **26.** $R(5, 1)$, $M(1, 4)$ **27.** $R(6, -2)$, $M(5, 3)$

28. $R(-7, 11)$, $M(2, 1)$ **29.** $R(4, -6)$, $M(-7, 8)$ **30.** $R(-4, -6)$, $M(3, -4)$

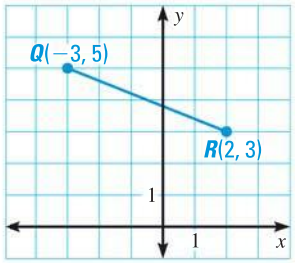
EXAMPLE 4 on p. 18 for Exs. 31–34

DISTANCE FORMULA **Find the length of the segment. Round to the nearest tenth of a unit.**

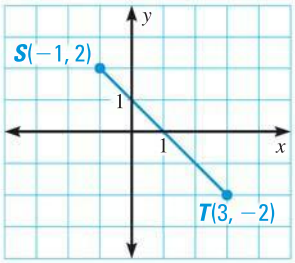
31.



32.



33.

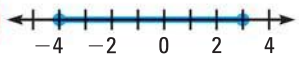


34. ★ MULTIPLE CHOICE The endpoints of $\overline{MN}$ are $M(-3, -9)$ and $N(4, 8)$. What is the approximate length of $\overline{MN}$?

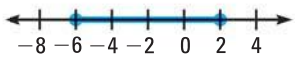
Ⓐ 1.4 units Ⓑ 7.2 units Ⓒ 13 units Ⓓ 18.4 units

NUMBER LINE **Find the length of the segment. Then find the coordinate of the midpoint of the segment.**

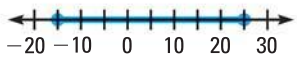
35.



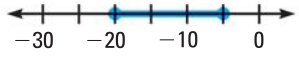
36.



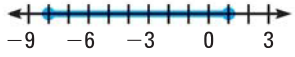
37.



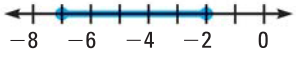
38.



39.



40.



41. ★ MULTIPLE CHOICE The endpoints of $\overline{LF}$ are $L(-2, 2)$ and $F(3, 1)$. The endpoints of $\overline{JR}$ are $J(1, -1)$ and $R(2, -3)$. What is the approximate difference in the lengths of the two segments?

Ⓐ 2.24 Ⓑ 2.86 Ⓒ 5.10 Ⓓ 7.96

42. ★ SHORT RESPONSE One endpoint of $\overline{PQ}$ is $P(-2, 4)$. The midpoint of $\overline{PQ}$ is $M(1, 0)$. *Explain* how to find PQ.

COMPARING LENGTHS **The endpoints of two segments are given. Find each segment length. Tell whether the segments are congruent.**

43. $\overline{AB}$: $A(0, 2)$, $B(-3, 8)$
$\overline{CD}$: $C(-2, 2)$, $D(0, -4)$

44. $\overline{EF}$: $E(1, 4)$, $F(5, 1)$
$\overline{GH}$: $G(-3, 1)$, $H(1, 6)$

45. $\overline{JK}$: $J(-4, 0)$, $K(4, 8)$
$\overline{LM}$: $L(-4, 2)$, $M(3, -7)$

46. xy ALGEBRA Points S, T, and P lie on a number line. Their coordinates are 0, 1, and x, respectively. Given $SP = PT$, what is the value of x?

47. CHALLENGE M is the midpoint of $\overline{JK}$, $JM = \frac{x}{8}$, and $JK = \frac{3x}{4} - 6$. Find MK.

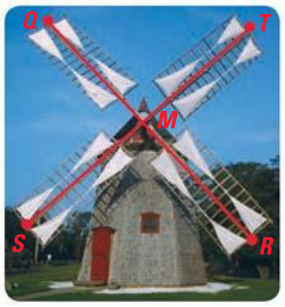
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

PROBLEM SOLVING

EXAMPLE 1 on p. 15 for Ex. 48

48. WINDMILL In the photograph of a windmill, $\overline{ST}$ bisects $\overline{QR}$ at point M. The length of $\overline{QM}$ is $18\frac{1}{2}$ feet. Find QR and MR.



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49. DISTANCES A house and a school are 5.7 kilometers apart on the same straight road. The library is on the same road, halfway between the house and the school. Draw a sketch to represent this situation. Mark the locations of the house, school, and library. How far is the library from the house?

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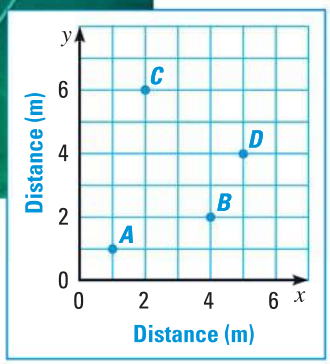
ARCHAEOLOGY The points on the diagram show the positions of objects at an underwater archaeological site. Use the diagram for Exercises 50 and 51.



50. Find the distance between each pair of objects. Round to the nearest tenth of a meter if necessary.

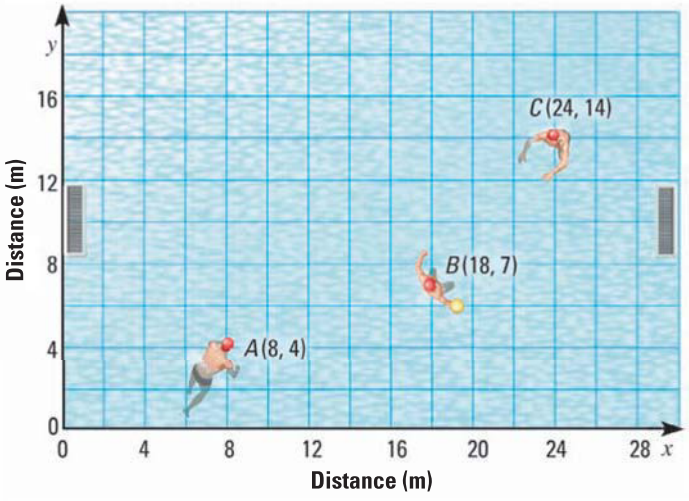
a. A and B **b.** B and C **c.** C and D

d. A and D **e.** B and D **f.** A and C

51. Which two objects are closest to each other? Which two are farthest apart?

Animated Geometry at classzone.com

52. WATER POLO The diagram shows the positions of three players during part of a water polo match. Player A throws the ball to Player B, who then throws it to Player C. How far did Player A throw the ball? How far did Player B throw the ball? How far would Player A have thrown the ball if he had thrown it directly to Player C? Round all answers to the nearest tenth of a meter.

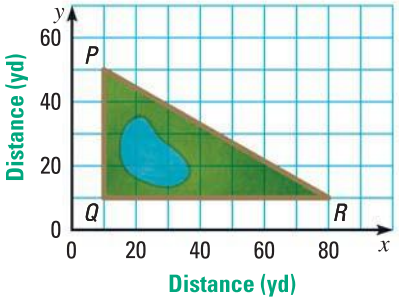


53. ★ **EXTENDED RESPONSE** As shown, a path goes around a triangular park.

a. Find the distance around the park to the nearest yard.

b. A new path and a bridge are constructed from point Q to the midpoint M of $\overline{PR}$. Find QM to the nearest yard.

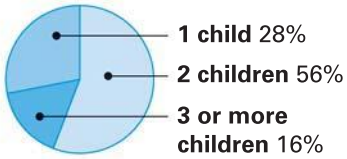
c. A man jogs from P to Q to M to R to Q and back to P at an average speed of 150 yards per minute. About how many minutes does it take? *Explain*.



54. **CHALLENGE** $\overline{AB}$ bisects $\overline{CD}$ at point M, $\overline{CD}$ bisects $\overline{AB}$ at point M, and $AB = 4 \cdot CM$. *Describe* the relationship between AM and CD.

MIXED REVIEW

The graph shows data about the number of children in the families of students in a math class. *(p. 888)*



55. What percent of the students in the class belong to families with two or more children?

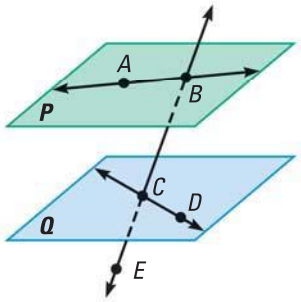
56. If there are 25 students in the class, how many students belong to families with two children?

PREVIEW
Prepare for Lesson 1.4 in Exs. 57–59.

Solve the equation. *(p. 875)*

57. $3x + 12 + x = 20$ **58.** $9x + 2x + 6 - x = 10$ **59.** $5x - 22 - 7x + 2 = 40$

In Exercises 60–64, use the diagram at the right. *(p. 2)*



60. Name all rays with endpoint B.

61. Name all the rays that contain point C.

62. Name a pair of opposite rays.

63. Name the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$.

64. Name the intersection of $\overleftrightarrow{BC}$ and plane P.

QUIZ *for Lessons 1.1–1.3*

1. Sketch two lines that intersect the same plane at two different points. The lines intersect each other at a point not in the plane. *(p. 2)*

In the diagram of collinear points, $AE = 26$, $AD = 15$, and $AB = BC = CD$. Find the indicated length. *(p. 9)*

2. DE **3.** AB **4.** AC

5. BD **6.** CE **7.** BE

8. The endpoints of $\overline{RS}$ are $R(-2, -1)$ and $S(2, 3)$. Find the coordinates of the midpoint of $\overline{RS}$. Then find the distance between R and S. *(p. 15)*